

KAPREKAR CONSTANT 6 1 7 4.

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Sixty years after its discovery and twenty years after Kaprekar’s death, Kaprekar Constant 6174 continues to fascinate amateur mathematicians, mathematics teachers and devotees of recreational and experimental mathematics.

Here is a common problem in elementary school arithmetic. Given a number, use its digits to write down the largest and the smallest possible numbers. For example, if the given number is 2008, the largest possible number is 8200 and the smallest is 0028. This simple problem underlies a fascinating adventure in recreational mathematics (“Maths for Fun”) leading to the magical number 6174.

Kaprekar Constant 6174.

Try the following simple arithmetical calculations.

(A) Pick a 4-digit number.

(B) Using the 4 digits write the largest and the smallest possible numbers. Find their difference. If the result is less than 4 digits, add leading 0’s to make it a 4 digit number.

(C) Repeat (B) on the 4-digit result above.

We illustrate by an example:

(A) Pick 2008

(B) The largest and the smallest numbers are 8200 and 0028. The smallest number is obtained by simply reversing the largest number. (Can you see why?) Their difference is

$$8200 - 0028 = 8172.$$

(C) Using the digits of 8172, the largest and the smallest numbers are 8721 and 1278. Their difference

$$8721 - 1278 = 7443.$$

Using (B) repeatedly $7443 - 3447 = 3996$

$$9963 - 3699 = 6264$$

$$6642 - 2466 = 4176$$

$$7641 - 1467 = \mathbf{6174}$$

$$7641 - 1467 = 6174$$

We can stop here because all subsequent steps will only give 6174. 6174 is the end-point of our calculation. It is called a **fixed point**.

6174 is called the **Kaprekar Constant**. It was discovered in 1946 by an Indian mathematician D.R. Kaprekar (1905-88) who worked as a schoolteacher in a small town Devlali in Maharashtra. The number is remarkable for two reasons.

(1) Whatever is the chosen 4-digit number at start [(A)], the terminal fixed point is always 6174. The only exceptions – to be avoided – are the nine numbers 1111, 2222 9999 in which all the four digits are the same. (Can you see why?)

(2) Further, the fixed point is reached in no more than 7 subtractions or steps. An example of a starting number requiring only one subtraction is 2583. The largest and the smallest numbers are 8532 and 2358 and their difference $8532 - 2358 = 6174$. The number 3538 requires 7 steps - the maximum number – to reach 6174; the results of successive subtractions are:

$$3538 \rightarrow 5175 \rightarrow 5994 \rightarrow 5355 \rightarrow 1998 \rightarrow 8082 \rightarrow 8532 \rightarrow 6174.$$

Kaprekar discovered 6174 in 1946 but took three years to convince himself that 6174 is the only possible fixed point. Even as a child Kaprekar was fond of puzzles and played with numbers. His name is associated with several interesting properties of numbers; besides Kaprekar Constant, there are Kaprekar Number (different from Kaprekar Consant), self-numbers (or Devlali numbers), Harshad numbers, Kaprekar Magic Square etc. Kaprekar

must have been familiar with the problem of largest and smallest possible numbers mentioned at the beginning, but he went further and asked what happens if their difference is taken and the process of sorting, reversing and subtracting – called the **Kaprekar Routine** – is repeated? Kaprekar Routine is nothing but the item (B) in para 2 above.

All 4-digit numbers subjected to Kaprekar Routine terminate at 6174 but the **number of iterations** (subtractions) required is a number 1 to 7, depending on the starting number. So every 4-digit number has a **Kaprekar Iteration Number (KIN)**.

“Maths for Fun”

Every calendar date can be transformed into a 4-digit number. One way to do it is the following. The date of our country’s Independence 15-08-1947 can be written as two numbers 1508 (date-month) and 1947 (year). They are added to get $1508 + 1947 = 3455$. Starting with 3455, we require only three iterations or steps to reach 6174:

$$3455 \rightarrow 2088 \rightarrow 8532 \rightarrow 6174.$$

So our Independence Day has $KIN = 3$.

As another example: Kaprekar’s birthday (17-01-1905) gives a starting number $1701 + 1905 = 3606$. You can easily verify his $KIN = 3$.

Of course KIN depends on the format adopted for the date. We have used the format commonly used in India (Day-Month-Year). Every calendar date is also associated with a day of the week 1 to 7 (Mon = 1, Tue = 2 Sun = 7), just like KIN. Kaprekar’s birthday was Tuesday (2). So, every calendar date is associated with two numbers, which have values 1 through 7. Occasionally, the two numbers – the KIN and the week-day - may be equal.

A ‘google’ search on ‘Kaprekar’ yields several thousand hits. Kaprekar fixed points for n-digit numbers (n upto 50) have been found using fast computers by devotees of experimental and recreational mathematics.

For $n = 3$ the only fixed point is 495 (check $954 - 459 = 495$). The only fixed point for $n = 4$ is 6174. In some instances Kaprekar Routine terminates not in a single number (like 6174) but in a **loop** or a cycle. For example for $n = 2$, all starting numbers will enter a loop (09→81→63→27→45→09) of length 4. For $n = 5, 7$ there are no fixed points, but only loops. For large n in general, there are many fixed points as well as cycles. For $n = 6$ there are two terminating fixed points (549945 and 631764) and one loop of length 7—in other words some starting numbers terminate in 549945, some others in 631764 and the rest enter the loop. For $n = 10$, there are three fixed points and five different loops. The numbers of fixed points and loops increase rapidly as the size of the number (n) is increased. Large numbers beyond $n = 50$ can be explored only with the fastest computers and one has to depend on mathematical analysis.

Readers – especially students – are urged to discover for themselves the Kaprekar Constant 6174. The arithmetic involves only subtraction of 4-digit numbers and it requires no more than 7 subtractions to reach 6174. So no calculator is needed; paper and pencil will do.

To start with, what is your KIN ?

(To learn more about Kaprekar Constant 6174, google on ‘Kaprekar Constant’. The article “The Mysterious number 6174” by Yutaka Nishiyama is highly recommended.)

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