P r o l o g u e

WORD PUZZLES AND THE LAWS OF CHANCE

An article with the above title was written in 1956, about the same time I was writing my PhD thesis in the Tata Institute of Fundamental Research (Bombay). It is reproduced here (August 2012) with some minor changes (p4-12).

In early 1950's some magazines introduced a new type of crossword puzzle. Unlike the regular crossword puzzle, the new puzzle had several clues, each with two answers both equally valid. The answer could be decided by flipping a coin. That was how the 'solvers' filled the puzzle grid and submitted it to the magazine publisher, paying a fee. This brand of crossword puzzles became a *word puzzle lottery* instead of a pastime testing linguistic skills.

A large sum was offered as prize money for all correct (0-error) entries and lesser prizes for 1,2,3-error entries. The total number of entries could have been as high as a million. A debate ensued, whether the puzzle was a camouflage for a gigantic lottery or a genuine game of skill. Since gambling was illegal, the matter called for Government action.

I remember the popular magazine '*The Illustrated Weekly of India*' publishing these puzzles. Composing the puzzles required lot of skill: to fit dual-solution words (both across and down) in the interlocking grid. The most popular Tamil weekly of the time, '*Ananda Vikatan'* also carried similar Tamil crossword puzzles regularly. Some later entrepreneurs discarded the cumbersome grid structure and simply listed the clues without any reference to a crossword grid. A typical puzzle had 18 clues, each with two possible answers to choose.

If the puzzles were filled by random choice of answers, well-known laws of probability should enable prediction of 0,1,2,3-error entries in the large sample of entries submitted. If it conformed to the announced results, then the hypothesis of coin tossing as the strategy of the solver, becomes credible. The above analysis was done for 12 puzzles and the results supported the hypothesis. The purpose of the article was to explain to laypersons, how laws of chance applied here. The approach was intuitive and by deliberate choice, no equations appear in the article. Readers who are familiar with elementary probability theory can skip major portions of the article (especially sections 1,2). This article was submitted to the daily newspaper *The Hindu* in 1956 and was rejected. I learnt that the word puzzle lottery was mainly an Indian innovation; it was not known in Europe or the US. Winning a prize was very rare. Yet I knew a person who won the Grand Prize in a puzzle from '*Ananda Vikatan'*. The prize amount of about Rs. 20,000 was shared with four others; the group of five had jointly submitted a large number of entries to enhance the chances of winning. The person was an elderly family relative.

What trigged my interest in this topic? In early 1950's, appeared an encyclopedic work in four volumes – '*The World of Mathematics'* by Prof. James R. Newman, an editor of the popular American magazine 'Scientific American'. It contained pioneering original articles by renowned authors covering a wide range of applications of mathematics and statistics, across many disciplines in Science, especially Behavioral Sciences (Economics, Sociology). Prof. Newman prefaced each article with insightful commentary. There were for example articles like "Statistics of Deadly Quarrels" (L.F. Richardson), 'Classification of Men according to their natural gift' (Sir Francis Galton). I spotted the book, not in T.I.F.R (Bombay) where I worked, but in Khallikote College Library in my native town Berhampur (Orissa), while on a vacation. My father, the Head of the Dept. of Mathematics had ordered the book for the library.

I took up the investigation of word puzzles as lotteries, as a sociological study of human behavior in word puzzle solving – to decide, if possible between two strategies: "intelligent choice" or "random chance".

Today this brand of puzzles does not exist in India (presumably it is prohibited by law). However the conventional crossword puzzles – of both the American and the British (cryptic) variety - continue to flourish all over the world as the most popular linguistic puzzles that are entertaining as well as intellectually challenging.

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9 August 2012

WORD PUZZLES AND THE LAWS OF CHANCE

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In the last few years, word puzzle lotteries have become very popular in our country. Prizes offered run into five figures. It is felt that these lotteries have a deleterious effect on the public and the Government has already brought forward a Bill to limit the magnitude of the prizes offered in these lotteries.

In a typical puzzle there are 18 clues, each with two possible answers. Solver chooses one of the answers.. All the 18 clues filled in with answers, constitutes an entry. A solver can submit any number of entries, paying a fee for each entry. Grand prize is for no-error entries. Smaller prizes are offered for 1-error, 2-error and 3-error entries.

It is generally believed that winning a word puzzle is almost entirely a matter of chance and "good luck." Some however, including the sponsors of these puzzles, contend that common sense and simple straightforward logic can be of help to win a fortune in word puzzles. The first hypothesis can readily be tested because where laws of chance alone decide the results, it is possible to predict them. For example, if all the competitors gambled in word puzzles, it is possible to calculate, using simple laws of probability theory, the number of entries with no errors, 1,2 and 3 errors, which are eligible for the prizes.

The purpose of this article is to explain how the laws of chance apply to the problem of word puzzles and to determine whether there is justification to suppose that the strategy of the participants in the puzzles is essentially gambling. Common sense and simple logic are certainly enough to understand the word puzzle as a game of chance, though not to solve it. Here, we are not concerned with the ethical aspect of word puzzles, their merits or demerits. *This article may be regarded as a study of human behavior in solving word puzzles.*

Let us describe the problem we propose to solve: We have a word puzzle containing 18 clues. For each clue there are two letters as possible answers. Clues are numbered 1 to 18, each with an empty square to fill. With all the empty squares filled in, we have an *entry*. *We assume that for each clue both solutions are equally probable.*

(A) First, we want to find the total number of different entries possible.

(B) Next, how many of these entries will contain only one error, how many will contain two errors, three errors and so on.

1. How many different entries are possible?

Answer to (A) is easily obtained as follows. Let us look at the first clue. We are asked to decide what is essential for a happy life: HEALTH or WEALTH. The answer is H or W. The second clue reads: for proper upbringing, child needs care of PA or MA. The choice here is either P or M. For each of the two above clues, if both the answers are equally plausible then there are four different ways (2 x 2) of filling the first two answers. They are

H	W	H	W
P	P	M	M

Let us suppose that the answer for the third clue needs either L or T. For every one of the above 4 ways in which the first and second answers can be filled, we have 2 ways of filling up the third. Thus we have 2 x 2 x 2 or 8 and only 8 different ways of filling the first three answers. They are:

H	W	H	W	H	W	H	W
P	P	M	M	P	P	M	M
L	L	L	L	T	T	T	T

Extending the same argument up to the last (18^{th}) clue, we have 2 x 2 x 2.......x 2 (2^{18}) or 262,144 and *only* 262,144 different possible entries to fill for a word puzzle with 18 clues. Only one out of them will match the "correct" solution decided upon by the sponsor. Since we have assumed that for every clue both the solutions are equally likely, it follows that every one of the 262,144 entries has the same chance of turning out to be the correct solution.

2. Probability of entries with no errors, 1,2,3 errors.

Answer to question (B) – to find the number of entries with 1, 2, 3 ..... errors is not as easy as obtaining the total number of possible entries. It is desirable to adopt a different approach. It is intuitive and uses the language of probability.

Suppose there is a bag containing a large number of white balls and an *equal* number of black balls. Without seeing the contents, you are asked to draw one ball at a time. You are allowed two draws. What is the probability that you will get a pair of black balls ?

In the first draw, the probability of obtaining a black ball is 1/2 and the probability of drawing a white ball is also 1/2 since white and black balls are equal in number. The probability that you will draw *either* a black or a white ball is the *sum* of individual probabilities: 1/2 + 1/2 =1. Unit probability means "certainty". The concept of probability is intuitive. (For example, if there are 10 white balls and 90 black balls in the bag, the probability of drawing a black ball is 10/(10+90) = 10 %).

The probability of drawing a black ball in the second attempt is again 1/2 (assuming that the total number balls in the bag is so large, that removal of any ball in the first attempt, does not affect appreciably the relative probabilities for drawing a white or black ball). For drawing black balls in both the attempts, the probability is given by the *product* of the individual probabilities: 1/2 x 1/2 =1/4. That is, the probability of getting a black pair is 1/4. The probability of getting a white pair is also 1/4. Now, we ask what is the probability of drawing a white ball and a black ball? This mixed pair can be obtained in two different ways: (1) a black ball followed by a white ball or (2) a white ball followed by a black ball. Each of these "events" has probability 1/4. Therefore, the probability of a mixed pair is 1/4 + 1/4 = 1/2. It can be verified that getting a black pair, or a white pair or a mixed pair is a certainty. The sum of the probabilities 1/4 +1/4 + 1/2 = 1.

The above example illustrates the two basic laws of probability, which are: if there are two *independent* "events" A and B, each taking place with a certain probability, then the probability that *either A or B* will happen is equal to the sum of the individual probabilities. This is called the *additive law* of probability. The probability that *both "events" A and B* will happen is the product of the individual probabilities. This is the *multiplicative law* of probability. The term "independent" qualifying the events means that event A is not influenced by event B and vice versa.

We can now recalculate the probability of a particular entry being an all-correct entry as follows. The probability that the answer to the first clue is correct is 1/2. The probability that the answer to the second clue is correct is 1/2. According to the multiplicative law of probability, the probability that answers to both the clues are correct is 1/2 x 1/2 = 1/4. Extending the argument up to the 18^{th} clue, we have the probability that the answers to every one of 18 clues is correct is 1/2 x 1/2 x.........1/2 (18 times) or $(1/2)^{18}$ or 1/262,144.

Consider the case of Mr. Gambler, filling an entry in the following way. To fill the first answer he tosses a coin. He decides to fill in H, if the coin turns head up and W if tail turns up. In a similar way he fills all the 18 answers and completes an 'entry'. This entry has a probability of 1 in 262,144 to be the correct solution. If he fills up, for example, 15 more entries (18 clues in each entry) in the same way, then the probability of at least one of the 16 entries being the correct solution is 16 times greater, or 1 in 16,384. (To be absolutely certain of winning an all-correct prize, he needs to send all the possible 262,144 entries *without resort to coin tossing*. At the rate of 8 entries per rupee, the cost will be over Rs. 30,000).

What is the probability that a given entry will contain only *one error* ? We proceed as follows: the probability that the answer to the first clue is wrong and the answers to all the other 17 clues are right is 1/2 x 1/2 x.....1/2 or $(1/2)^{18}$. The probability that the second answer alone is wrong and all the other answers are correct is also $(1/2)^{18}$. Therefore the probability that *any one* of the 18 answers is wrong and the other answers are correct is $(1/2)^{18}$ +$(1/2)^{18}$ +$(1/2)^{18}$ +.......(18 times) = 18 x $(1/2)^{18}$. This means that the probability of an entry containing one error is 18 times the probability of its containing no error. The probability that the entry will contain only one right answer (or 17 errors) is also the same.

What is the probability that an entry will contain *two errors*? Now it is obvious that the probability of any two chosen answers (say answers to clues 1 and 7) being both wrong and all the rest of the 16 answers being correct is $(1/2)^{18}$. But any two of the 18 answers may be wrong for a two-error entry. Therefore to obtain the probability that an entry will contain two errors, $(1/2)^{18}$ has to be multiplied by a number that is equal to the number of different ways in which one can pick two clues from 18 clues. We show that this number is (18 x 17)/2 or 153.

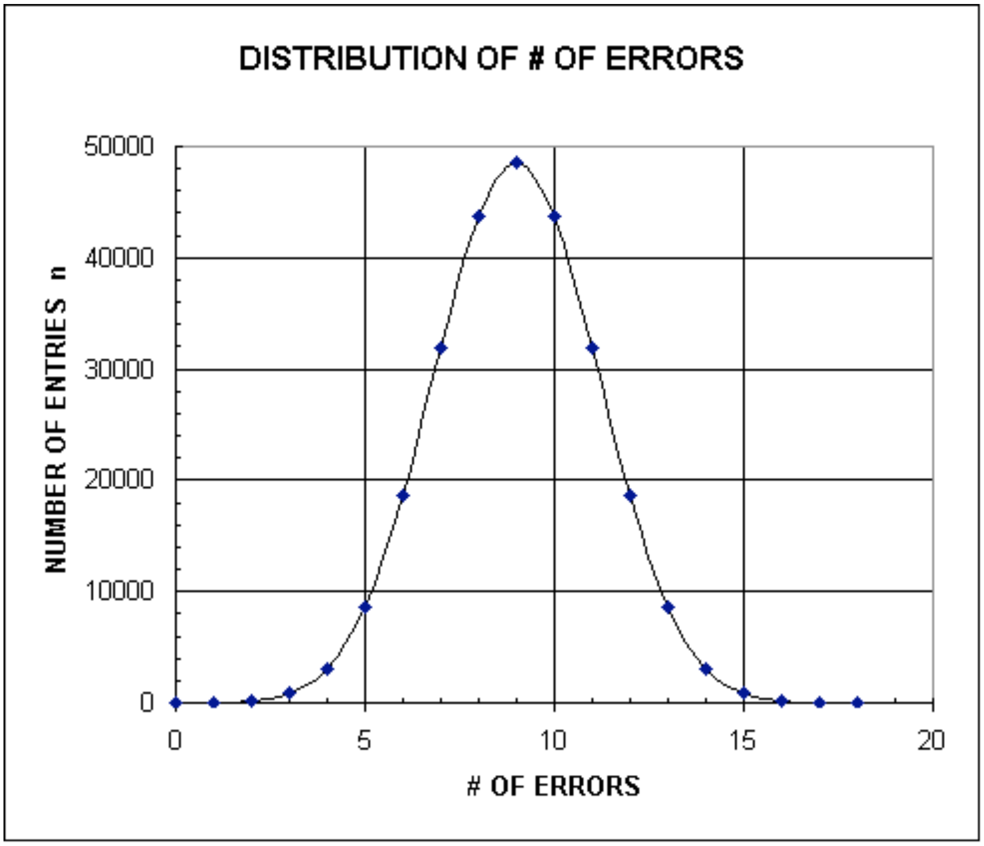
From 18 clues, the first clue can be chosen in 18 different ways (the total number of clues). Once the choice is made, there are only 17 clues left for the second choice and it can be made in 17 ways. This is true for every one of the 18 first choices and so there are in all 18 x 17 ways of choosing two different clues from 18 clues. However in the above procedure, a combination 1,7, for example, is counted as different from the combination 7,1. When all the 18 x 17 possibilities are written down, one will find such systematic repetition with the order of the two numbers reversed.. But for our analysis a combination 1,7 is the same as 7,1. So, there are only (18 x 17)/2 *distinc*t combinations of two clues chosen from 18. Thus, the probability of an entry containing two errors is 153 times the probability of its having no errors (=153/2622144).

To summarize, of the 262,144 possible entries, the number with 0,1,2 errors are 1, 18 and 153 respectively. The entire calculation of the numbers of entries with errors 0 to 18 is given in Table 1. The same data is also presented in Figure 1. Note the figure is symmetric and bell-shaped. The distribution of x, the number of errors, is called the ‘Binomial Distribution”.

Table 1

X	n(X)
0	1
1	18
2	153
3	816
4	3060
5	8568
6	18564
7	31824
8	43758
9	48620
10	43758
11	31824
12	18564
13	8568
14	3060
15	816
16	153
17	18
18	1
TOT	262144

Figure 1



An interesting and perhaps non-intuitive feature of Table 1 is that the number of entries with x errors is the same as the number with x correct answers. For example the number of entries with 2 errors and the number with 2 correct answers (or 16 errors) are equal (153). This is true only when both the answers to a clue are equally probable.

Comparison of observed results with expected results.

Let us assume that among the total number of entries submitted, there are equal numbers of entries favoring each of the two answers to any clue. Then the results above are directly applicable. One expects the number of entries with 0,1,2,3.... errors to increase in the proportion 1, 18, 153, 816.......

Results are available for 12 puzzles (each containing 18 clues) from an Indian magazine. The numbers of no error, 1-error, 2-error and 3-error prize-winning entries are given for each puzzle. No data is available on numbers of entries with errors exceeding 3 and the total number of entries submitted. The data can be compared with expected numbers based on the hypothesis that all the entries are filled by a coin-tossing procedure. Results are given for three puzzles that are "typical" of the 12. See Table 2 below.

Table 2

x \ n(x)	Puzzle A			Puzzle B			Puzzle C		
	O	E	0-E (% DIFF)	O	E	0-E (% DIFF)	O	E	0-E (% DIFF)
0	3	3	0 (0)	3	4	-1 (-30)	11	17	-6 (-36)
1	59	59	0 (0)	77	76	1 (1.3)	270	308	-38 (-12)
2	517	503	14 (2.7)	720	644	76 (12)	2558	2614	-56 (-2.1)
3	2671	2685	-14 (-0.5)	3360	3436	-76 (-2.2)	14040	13940	100 (0.7)
N	**862,316**			**1,103,764**			**4,478,470**		

x = # of errors. n(x) = # of entries with x errors.
In each puzzle, there are three columns.
Column 1 is # of Observed (O) entries (published). Column 2 is the # expected (E) based on the Hypothesis. Column 3 is the difference (O-E) and the % difference in brackets. N is the estimated total number of entries.

Although the total number of entries N (sample size) is not published, we can estimate it as follows. From Table 1 we find the sum of entries with errors 0,1,2,3 is 988, out of possible 262,144 entries. The probability of up to 3 errors is therefore 988/262,144 or 1/265.328. Consider Puzzle A. The observed total number of errors up to 3 is 3+59+517+2671 = 3250. If N is the estimated sample size, the expected number of entries with errors up to 3, is N x (1/265.328). Equating this to the observed number 3250, we obtain N = 3250 x 265.328 = 862,316. The estimated sample sizes are given in the last row, Table 2. The procedure is called "normalizing" the observed and calculated results to be equal for the number of entries with errors up to 3.

Since the estimated sample size (N) is known, the individual numbers of entries with expected number of errors 0,1,2,3 can be calculated (See Table 2). For example, in puzzle (A), the expected number of 0-error entries is 862316 x (1/262144) = 3.3; the number of 1-error is 862316 x (18/262144) = 59.2.

The observed (O) and expected (E) values are strikingly similar, the agreement being best in puzzle A. (The expected values are rounded off to the nearest integer). The two values are identical for n(0) and n(1) – 3 and 59 respectively - and differ by a small percentage for n(2) and n(3). Numbers for puzzle B show a greater difference for n(2) and n(3). Puzzle C shows larger deviations of n(0) and n(1). It should be noted that the expected numbers are the *most probable* numbers and deviations, inherent in the random nature of the data, are to be expected. For example, if an unbiased coin is tossed 100 times, on an average one expects 50 heads and 50 tails. But actually one 100-toss experiment, may yield 60 heads and 40 tails. If the 100-toss experiment is repeated many times, the average number of heads and tails will be closer to 50 each.

Looking at all the 12 puzzles together, the most significant difference between the observed and expected numbers is for n(0). In 6 out of 12 puzzles the observed number is 1, compared to expected number about 4. For all the 12 puzzles the total numbers for n(0) (no-error entries) are 43 (observed) and 83 (expected). The range of estimated sample sizes is large, from about 862,000 (puzzle A) to about 4.9 million. If the deviation of observed number from the expected is due to a random process, it is equally likely to be positive or negative. This is the case for n(2) and n(3). But for n(0) and n(1), there are more negative deviations (O < E) than positive. More detailed analysis of data is not justified because of lack of data for errors greater than 3.

A solver may submit a large number of entries for a puzzle to increase the chances of winning a prize. But this is irrelevant for our analysis, since all entries are considered independent and prizes are awarded to the entries. It is possible, though unlikely that a solver who has submitted a large number of entries will have more than one entry with 3 errors.

3. Summary and Conclusion.

The word puzzle we have considered contains 18 clues, each with two possible answers. A solver submits an entry with all the 18 choices made. Prizes are awarded for 0,1,2,3- error entries. Data is available for 12 puzzles, listing the number of entries with errors up to 3 in each puzzle.

We wish to test the hypothesis that the entries are filled by solvers at random as in a coin-toss experiment. It is assumed that both the answers to a clue are equally probable. Then, the Binomial distribution gives the relative number of 0,1,2,3-error entries as 1:18:153:816. The numbers are available for 12 puzzles, but results are presented only for three (Table 2), which are representative of the kinds of differences between the observed (published) and expected (random choice) numbers. The general trend in the proportion of 0,1,2,3 errors is very similar for both observed and expected numbers. However, for 0 and 1-error entries, there seems to be a systematic trend towards observed being lower than the expected numbers. For example in all the 12 puzzles put together, there are 43 observed no-error entries against an expected number of 83. The corresponding figures for n(1) are 1353 and 1502. For n(2) they are 12,826 and 12749 and for n(3) they are 68124 and 68011. Given the limited statistics (data only for 0,1,2,3 errors), any further detailed analysis is not justified.

However we can conclude the following: the published data support the hypothesis that *a predominant majority of the entries submitted for the word puzzles, are filled by the solvers not by intelligent choice but by random chance. The word puzzle is regarded more as a lottery than as a test of linguistic skills.*

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(This unpublished article was written sometime in 1956)