Fibonacci-*Kolams*

S. Naranan

Abstract: This website demonstrates with theory and examples the connection between the beauty of Indian *Kolam*s with the elegance of the well known Fibonacci Numbers.

Prof S. Naranan, a Physicist, in course of drawing these *Kolams* as part of his many hobbies, discovered the elegant connection and has come up with some fascinating designs. The articles and actual drawings of the *Kolams* can be found by clicking the links on the left margin.

Introduction

Kolams are decorative geometrical patterns that adorn the entrances of households and places of worship especially in South India. *Kolam* is a line drawing of curves and loops around a regular grid of points. Usually *kolams* have some symmetry (e.g. four-fold rotational symmetry). There are variants:

(1) Lines without dots
(2) Lines connecting dots and
(3) Free geometric shapes without lines or dots.

The last variety has sometimes, brilliant colors and is known as *Rangoli,* popular in North India. The traditional South Indian *kolam*, based on a grid of points is known as *PuLLi Kolam* or *NeLi Kolam* in Tamil Nadu (*PuLLi* dot, *NeLi* = curve); *Muggulu* in Andhra Pradesh, *Rangavalli* in Karnataka and *Pookalam* in Kerala.

Special *kolams* are drawn on festive occasions with themes based on seasons, nature (flowers, trees) religious topics and deities. Large *kolams* with bewildering complexity are common on such occasions. The folk-art is handed down through generations of women from historic times, dating perhaps thousand years or more. Constrained only by some very broad rules, *kolam* designs offer scope for intricacy, complexity and creativity of high order, nurtured by the practitioners, mostly housemaids and housewives, both in rural and urban areas.

Here we deal only with *PuLLi Kolam* or *kolam* for short. The contents are organized in three sections: articles, *kolam* slides and special *kolams*.

The Articles:

- *KOLAM* DESIGNS BASED ON FIBONACCI NUMBERS.
  Part I: Square and Rectangular Designs
- *KOLAM* DESIGNS BASED ON FIBONACCI NUMBERS.
  Part II: Square and rectangular designs or arbitrary size based on generalized Fibonacci Numbers.

In Part I, we describe a general scheme for *kolam* designs based on numbers of the Fibonacci series (0, 1, 1, 2, 3, 5, 8, 13, 21, 34 …….). Square *kolams* (3^2, 5^2, 8^2, 13^2, 21^2) and rectangular *kolams* (2 x 3, 3 x 5, 5 x 8, 8 x 13) are presented. The modular approach permits extension to larger *kolams* and computer-aided design. This enhances the level of creativity of the art.

In Part II, the scheme is generalized to arbitrary sizes of square and rectangular *kolams* using generalized Fibonacci numbers. The problem of enumeration – the number of possible Fibonacci *kolams* of a given size – is discussed. Enumeration of *kolams* of small size (3^2, 5^2, 2 x 3) is described. For 2 x 3 *kolams* symmetry operators forming a group are used to classify them. Finally, the scheme is further extended beyond square grids to cover diamond-shaped grids. Possible connections of *kolams* to Knot theory and Group theory are indicated.

KOLAM SLIDES

This *PowerPoint* file has 49 slides. The first 16 slides contain interesting facts about Fibonacci Numbers. The remaining deal with the *Kolam* designs based on Fibonacci Numbers. The *kolams* are duplicated from the two articles mentioned above. Explanations for the *kolam* slides can be found in the articles.

SPECIAL *KOLAMS*

The Generalized Fibonacci Numbers permit a wide choice for the rectangles that go into the square designs. With the standard "Fibonacci Numbers" the rectangles are "Golden rectangles" with sides equal to consecutive Fibonacci numbers and their ratio approaching the Golden Ratio φ (=1.61803…). The explanations for the 6 slides are given below; *All these kolams have a single loop except the last one.*

1. **(7 4 11 15).** This code implies that a 15^2 *kolam* is made up of a central square 7^2 surrounded by four rectangles 4 x 11. The code for 7^2 is (3 2 5 7); or the 7^2 contains a central 3^2 surrounded by four rectangles 2 x 5. Each 4 x 11 rectangle is made up of three modules: 4^2, 4 x 3 and 4^2.

2. **(13 4 17 21).** Since 21 is a Fibonacci Number, the canonical quartet code for 21^2 is (5 8 13 21), The "Golden rectangles" are 8 x 13, two consecutive Fibonacci Numbers. But here we consider 21 as a "Generalized Fibonacci Number" and the quartet code is (13 4 17 21). The 21^2 is made up of a 13^2 surrounded by four rectangles 4 x 17. The 13^2 has the code (5 4 9 13) again different from the canonical (3 5 8 13). Each 4 x 17 rectangle is made up of 5 modules 4^2, 4^2, 4 x 1, 4^2, 4^2.

3. **(12 7 19 26).** This square *kolam,* we name it ***"a e-kolam"***, ***e*** representing the Euler's constant (2.7182818), the base for natural logarithms. The code implies that a 26^2 *kolam* has a central 12^2 enveloped by four rectangular *kolams* 7 x 19. The ratio of the sides 19/7 = 2.714.., differs from *e* by 0.004 or < 0.15 % of e. So this kolam has "*e-rectangles*" instead of the Golden rectangles. The central 12^2 has the code (8 2 10 12) – i.e. a central 8^2 surrounded by four rectangles 2 x 10. In turn the 8^2 has the canonical code (2 3 5 8). The 3 x 5s are Golden rectangles. So, this *kolam* features both the Golden rectangles and the e-rectangles.

4. **(7 19).** This rectangle of sides 7 x 19 with ratio approximately equal to *e*, the Euler's constant, was used in the in 26^2 *kolam* above. It is made up of three modules, a 7 x 5 rectangle sandwiched between two squares 7^2. The three modules are spliced together at 6 points, indicated by dots. If splices at the points A and B are added then the single loop *kolam* splits into three loops.

5. **(15 7 22 29).** This square *kolam,* we name it ***"a pi-kolam".*** The code (15 7 22 29) implies that 29^2 *kolam* contains a smaller 15^2 *kolam* at the center surrounded by four rectangles 7 x 22. The ratio of the sides of the rectangle 22/7 is the commonly used approximation for *pi*, the ratio of the perimeter to diameter of a circle.

- Now, 22/7 = 3.14285...differs from *pi* (=3.14159 . . .) by 0.0013, which is < 0.04 % of *pi.*

- The code for 15^2 is (7 4 11 15) and the code for 7^2 is (3 2 5 7). Each 7 x 22 rectangle is composed of 4 modules 7^2, 4 x 7, 4 x 7, 7^2 that are spliced together at 15 places to produce a 3-loop design.

- **This 29^2 *kolam* is built with 33 modules and 111 splicing points to produce a single loop traversing 841 (29^2) dots or grid points**.

6**. (7 22)**. This shows the 7 x 22 *"pi-rectangle*" used in the 29^2 square *kolam* above. The 15 splicing points come in three sets of 5 indicated by dots in the direction of arrows. The three loops are shown in different colours. It is possible to convert this into a single-loop *kolam* by removing the two splices at A and B.